MATHEMATICS METHODS UNITS 3 & 4

Semester Two

2016

SOLUTIONS

Calculator–free Solutions

1. (a) $\dfrac{d}{dx}[\ \ln(2x + 1) - 2x^{-2}\ ]$

$= \dfrac{2}{2x + 1} + \dfrac{4}{x^3}$ ✓✓

(b) $\dfrac{dx}{dt} = e^{2t} - \dfrac{1}{2}e^{t}$ ✓✓

(c) $f\,'(y) = 3\cos 3y + 4\sin(1 - 2y)$ ✓✓ [6]

2. (a) $(x + 2)\ln 3 = \ln 6$ ✓

$x = \dfrac{\ln 6}{\ln 2} - 2$ ✓

(b) $\ln x = 2\ln x + 2$ ✓

$\therefore \ln x = -2$ ✓

$\therefore x = \dfrac{1}{e^2}$ ✓

(c) $\dfrac{e^{\sqrt{x}} + e^{x}}{2} = e^{x}$ since $\dfrac{d}{dx}(e^{x}) = e^{x}$

$\therefore e^{\sqrt{x}} + e^{x} = 2e^{x}$

$\therefore e^{\sqrt{x}} = e^{x}$ ✓

$\therefore x = 0$ or 1 ✓ [7]

3. (a) $f\,'(x) = 2x\ln x + (x^2)\left(\dfrac{1}{x}\right) = 2x\ln x + x$ ✓

$\therefore x(2\ln x + 1) = 0$ ✓

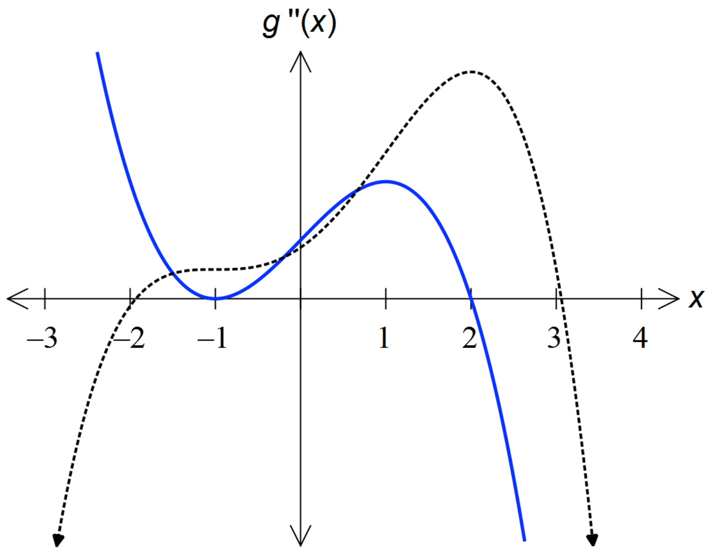
$\therefore \ln x = -\dfrac{1}{2}$ (disregard x = 0)

$\therefore x = \dfrac{1}{\sqrt{e}}$ ✓

$f\,''(x) = 2 + 2\ln x + 1$

$\therefore f\,''\left(\dfrac{1}{\sqrt{e}}\right) > 0 \therefore$ Min ✓

(b)



✓✓✓ [7]

4. (a) $\int \left[\frac{2}{x} + \sin\left(\frac{x}{2} + 3\right)\right] dx$

$= 2\ln x - 2\cos\left(\frac{x}{2} + 3\right) + c$ ✓✓

(b) $\left[\frac{x^3}{3} - e^{x+1}\right]_{-1}^{0}$ ✓

$= [0 - e^1] - \left[-\frac{1}{3} - 1\right] = \frac{4}{3} - e$ ✓✓

(c) $-2\tan 2x$ ✓✓ [7]

5. (a) Has only integer values for x. ✓

(b) $0 + \frac{1}{6} + \frac{1}{2} + \frac{1}{3} = 1 \therefore \Sigma f(x) = 1$ and $f(x) \geq 0$ for all x. ✓✓

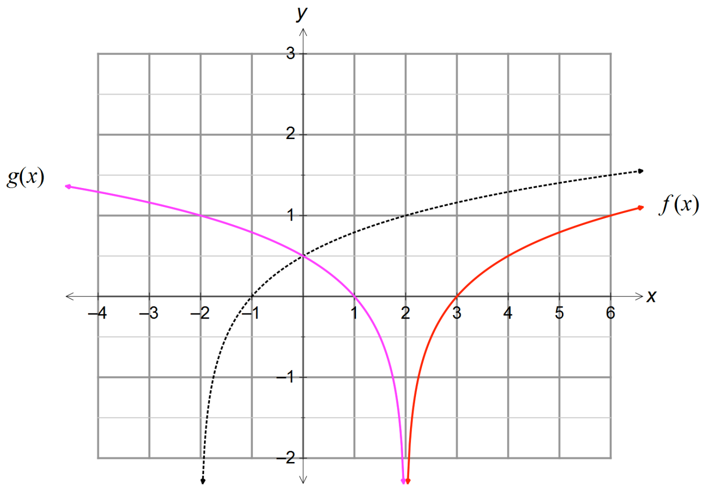
(c) (i) $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ ✓

(ii) $\frac{1}{6}$ ✓

(iii) $\frac{\frac{1}{2}}{\frac{4}{6}} = \frac{3}{4}$ ✓✓ [7]

6. (a) $c = 2$ and $b = 4$ ✓✓

(b)



✓✓✓✓ [6]

7. (a) Area is approximately 2 units^2 ✓

Use diagram or other to explain half of a 4 x 1 rectangle ✓

(b) (i) 2 ✓

(ii) –2 ✓

(iii) 2 ✓ [5]

8. (a) $\int_0^{\frac{\pi}{2}} 2\sin x \; dx$ ✓✓

(b) $\int_0^{0.67} (2\sin x - 1 - \cos 2x)\; dx + \int_{0.67}^{\frac{\pi}{2}} (\cos 2x + 1 - 2\sin x)\; dx$ ✓✓✓ [5]

Calculator–Assumed Solutions

9. (a) (i) $e^{\frac{1}{3}}$ ✓✓

(ii) $[-\ln e]^2 = 1$ ✓✓

(b) (i) $3000 = 2000e^k(1)$ ✓

$\therefore k = 0.4055$ ✓

(ii) $8000 = 2000e^{0{\cdot}4055t}$ ✓

$\therefore t = 3.419 \rightarrow$ 6·84 hours after 12*pm*

$\therefore$ 6:50 pm ✓ [8]

10. (a) $v = \pi\cos\pi t + c$ ✓

$(0, 0) \rightarrow c = -\pi$

$\therefore v = \pi\cos\pi t - \pi$ ✓

$\therefore x = \sin\pi t - \pi t + c$ ✓

$(0, 0) \rightarrow c = 0$ ✓

$\therefore x = \sin\pi t - \pi t$

(b) (i) $v = \pi\cos 2\pi - \pi = 0$ m/s ✓

(ii) $a = \pi^2$ ✓ [6]

11. (a) $\delta r = 0{\cdot}3r$ and $V = \frac{4}{3}\pi r^3$ ✓

$\therefore \delta V = \frac{dV}{dr} \times \delta r \rightarrow \delta V = 4\pi r^2 \times 0{\cdot}3r$ ✓

$\therefore \delta V = 1{\cdot}2\pi r^3$

$\therefore \frac{\delta V}{V} = \frac{1{\cdot}2\pi r^3}{\frac{4}{3}\pi r^3} = 0{\cdot}9$ ✓

Hence 90% increase. ✓

(b) (i) $x_A = t^3 - 2t^2 + 3t$ ✓

and $x_B = 2 - 3t - t^3$

$\therefore D = (t^3 - 2t^2 + 3t) - (2 - 3t - t^3) = 2t^3 - 2t^2 + 6t - 2$ ✓✓

(ii) $2t^3 - 2t^2 + 6t - 2 = 0$

$\therefore t = 0.36$ and $x = 0.87$ ✓✓ [9]

12. (a) $\int_0^{\frac{1}{2}} (ax^2 + 1)\ dx = 1$ ✓

$\therefore \left[\frac{ax^3}{3} + x\right]_0^{\frac{1}{2}} = 1$ ✓

$\therefore \frac{a}{24} + \frac{1}{2} = 1 \rightarrow a = 12$ ✓

(b) (i) $P(X < \frac{1}{4}) = \int_0^{\frac{1}{4}} (12x^2 + 1) = \frac{5}{16}$ ✓✓

(ii) $P(X < \frac{1}{8} \mid X < \frac{1}{4}) = \dfrac{P\left(x < \frac{1}{8}\right)}{\frac{5}{16}}$ ✓

$= \dfrac{\frac{17}{128}}{\frac{5}{16}} = \frac{17}{40}$ ✓

(c) $g(x) < 0$ for $x > 1$ ✓ [8]

13. (a) (i) $\frac{dA}{dt} = -16 + e^{1\cdot 6} = -11\cdot 05$ ✓✓

(ii) $f(t) = -t^2 + e^{0\cdot 4t}$ ✓

Min occurs when $t = 9.7$ ✓

$\therefore$ June 10th ✓

(b) $\int_0^{12} -t^2 + e^{0\cdot 4t}\ \mathrm{dt} = -274\cdot 7$ ✓✓

$\therefore$ decrease of 274.7 m^2 ✓

(c) Total $= 6000 + \int_0^{15} -t^2 + e^{0\cdot 4t}\ \mathrm{dt} = 5881\cdot 1\ m^2$ ✓✓ [10]

14. (a) (i) $v(t) = at(t-6)$ ✓

$(3, 6) \rightarrow a = -\frac{2}{3}$

$\therefore v(t) = -\frac{2}{3}t(t-6)$ ✓

(ii) $v(t) = 12 - 2t$ ✓

(b) $\int_0^6 -\frac{2}{3}\ t(t-6)\ \mathrm{d}t = 24$ ✓✓

(c) $\int_6^b (12 - 2t)\ \mathrm{d}t = -24$ ✓

$\therefore b = 10\cdot 9$ ✓ [7]

15. (a) $\bar{x} = 4{\cdot}5$ and $\sigma_x = \sqrt{4{\cdot}5(0{\cdot}55)} = 1{\cdot}57$ ✓✓

(b) (i) P(X = 5) = 0.2340 ✓✓

(ii) P(X ≤ 6) = 0.8980 ✓✓

(iii) $P\left(X \le 3 \mid X \le 6\right) = \dfrac{P(X \le 3)}{0{\cdot}8980}$

$= \dfrac{0{\cdot}2660}{0{\cdot}8980} = 0{\cdot}2962$ ✓✓ [8]

16. (a) $P(4 \le t \le 6 \mid t > 3)$

$= \dfrac{0{\cdot}1448}{0{\cdot}4724} = 0{\cdot}3064$ ✓✓

(b) $e^{-0{\cdot}25t} = 0{\cdot}5 \rightarrow t = 2{\cdot}77$ min ✓✓ [4]

17. (a) (i) $P(X < 164) = 0{\cdot}6554$ ✓✓

(ii) $P(161 < X < 163 \mid X < 164) = \dfrac{0{\cdot}1585}{0{\cdot}6554} = 0{\cdot}2418$ ✓

∴ 36 girls ✓

(b) $P(X > h) = 0{\cdot}9 \rightarrow h = 155{\cdot}6$ cm ✓✓

(c) $N(175, \sigma^2) \rightarrow N(0, 1) \rightarrow z = 0{\cdot}4307$ ✓

∴ $0{\cdot}4307 = \dfrac{180 - 175}{\sigma} \rightarrow \sigma = 11{\cdot}61$ ✓✓ [9]

18. (a) $\hat{p} = \dfrac{15}{59} = 0{\cdot}2542$ ✓

(b) $\hat{p} - 1{\cdot}96\sqrt{\hat{p} \times \dfrac{1-\hat{p}}{n}} \le p \le \hat{p} + 1{\cdot}96\sqrt{\hat{p}\,\dfrac{1-\hat{p}}{n}}$

∴ $0{\cdot}2542 - 0{\cdot}1111 \le p \le 0{\cdot}2542 + 0{\cdot}1111$ ✓✓

∴ $0{\cdot}1431 \le p \le 0{\cdot}3653$ ✓

(c) $0{\cdot}1111 = 11{\cdot}11\%$ ✓

(d) Increase the sample size. ✓

(e) $n = \hat{p}\left(1 - \hat{p}\right)\left(\dfrac{z}{E}\right)^2 = (0{\cdot}2542)(0{\cdot}7458)\left(\dfrac{1{\cdot}96}{0{\cdot}04}\right)^2$ ✓

∴ $n = 455.19$ ✓

∴ 456 cricketers [8]

19. (a) (i) $m + n = 0{\cdot}5$ and $30m + 40n = 17$ ✓

∴ $m = 0{\cdot}3$ and $n = 0{\cdot}2$ ✓

(ii) 0.2 + 0.1 = 0.3 ✓

(iii) $\dfrac{P(X > 30)}{P(X > 20)} = \dfrac{0{\cdot}4}{0{\cdot}7} = \dfrac{4}{7}$ ✓✓

(b) (i) 13.2 ✓

(ii) Old $\sigma_x = 12{\cdot}5$ ✓

New $s_x = 1{\cdot}25 \rightarrow V(x) = 1{\cdot}5625$ ✓ [8]

20. (a) A sample that reflects the whole population. ✓

(b) $\frac{54}{90} = 0{\cdot}6$ ✓

(c) $\hat{p} = 0{\cdot}6$

∴ 90% confidence interval

$= 0{\cdot}6 \pm 1{\cdot}645\sqrt{\frac{(0{\cdot}6)(0{\cdot}4)}{90}} = 0{\cdot}6 \pm 0{\cdot}085$ ✓✓

$= 0.515 \leq p \leq 0.685$ ✓✓

(d) (i) $\hat{p} = \frac{35}{50} = 0{\cdot}7$

Since not in the 90% confidence interval probably not in the cohort of Year 4 students. Maybe a higher grade. ✓

(ii) $\hat{p} = \frac{71}{120} = 0{\cdot}59$

90% confidence interval

$= 0.518 \leq p \leq 0.665$ ✓

∴ Can reasonably expect that the sample came from the Year 4 cohort, as this interval is within the bounds. ✓ [9]

21. (a) $a\int_0^e \frac{x}{x^2 + e^2}\,dx = \ln 2$

$\therefore \frac{a}{2}\left[\ln(x^2 + e^2)\right]_0^e = \ln 2$ ✓✓

$\therefore \frac{a}{2}[\ln(2e^2) - \ln(e^2)] = \ln 2$ ✓

$\therefore \frac{a}{2}\ln(2) = \ln 2$

✓

$\therefore a = 2$

(b) Convenience sampling, so is non-random. ✓

Bias: Houses without TVs

Interested group would be vocal

Age and gender bias to Channel 2 viewers ✓ [6]

END OF QUESTIONS